Total Number of Questions : 24

Time : 2.00 Hours

Max. Marks : 100

1. Prove : Characteristic of a field F is 0 then F is infinite. (3 Marks)

2. Explain : Is every field F has an infinite extension of F. (3 Marks)

3. Find the value of the improper integral (3 Marks)

   $\int_1^\infty \frac{1}{t^p}\,dt$ for $p \in \mathbb{R}$.

4. What is the cardinality of the set of all complex functions $f(z) = u(x, y) + i\,v(x, y)$, $z = x + iy$ such that u is a harmonic conjugate of v and v is a harmonic conjugate of u ? Justify. (3 Marks)

5. Find the remainder when $6^{n+2} + 7^{2n+1}$ is divided by 43. (3 Marks)

6. Prove : The center of the set of all $n \times n$ matrices over a field F is isomorphic to F. (4 Marks)

7. Find the number of similarity classes of idempotent matrices of order n over a field F. Explain your answer. (4 Marks)

8. Prove : S_5 is not solvable. (4 Marks)

9. Check whether the following sequence of functions $g_n(x) = \dfrac{1}{n(1+x^2)}$ converges uniformly or diverges on $\mathbb{R}$. (4 Marks)

10. Find the points on $\mathbb{R}^2$ where the directional derivative of the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = \sqrt{x^2 + y^2}$ exists. Also find the directional derivative(s) if it exists. (4 Marks)

11. Let f(z) be an entire function and M is a constant such that for a positive real number R and for an integer $n \geq 1$ $|f(z)| \leq M|z|^n$ for $|z| > R$. Prove that f(z) is a polynomial of degree less than or equal to n. (4 Marks)

12. Let X be a normed linear space, $S = \{x \in X / \|x\| \leq 1\}$ and f be a map from S into $\mathbb{R}$ such that $f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$ for all $x, y \in S$ and $\alpha x + \beta y \in S$ where α and β are scalars. Is there is an extension for f to all of X ? Justify. (4 Marks)

13. Let X, Y, Z be topological space. If $f : X \to Y$ and $g : Y \to Z$ are continuous then show that $g \circ f : X \to Z$ is continuous. (4 Marks)

14. A metric space X is connected iff every continuous function $f : X \to \{0, 1\}$ is not onto. (4 Marks)

15. Prove or disprove if A and C are connected subsets of a metric space X and if $A \subseteq B \subseteq C$ then B is connected. (4 Marks)

P.T.O.